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CS 383
Final Exam
December 13, 2017

The exam has 8 questions. #7 is worth 16 points; the other seven are worth 12 points each

1. Identify the seven languages below as
   - R = regular
   - C = context-free but not regular
   - D = recursive (decidable) but not context-free
   - E = recursively enumerable but not recursive
   - N = not recursively enumerable

   You don't need to justify your answers.

   a. $\{ (01)^n \mid n >= 0\}$ For example, 010101 is in this language

   b. $\{(0^n1)^n \mid n>0\}$ For example, 000100010001 is in this language.

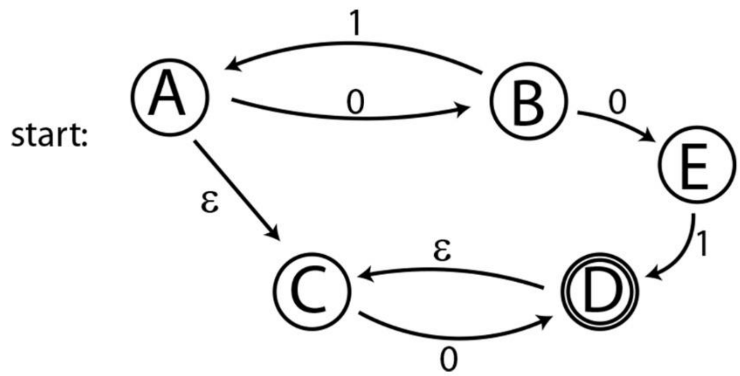
   c. Strings of the digits 0 thru 9 where no digit appears more than two times.

   d. Strings of the form *ww*, where *w* is a string of 0s and 1s.

   e. {m | m is a valid encoding of a Turing Machine} (Remember that we encoded a transition $\delta(q_i,t_j)=(q_k,t_L,d_m)$ as $0^i10^j10^k10^L10^m1$ and encoded the TM as a sequence of transitions followed by the final state).

   f. The set of encodings of Turing Machines that *do* accept their own encodings. Don't confuse this with the diagonal language, which is the set of TMs that *don't* accept their own encodings.

2. Here is an ε-NFA with A as its start state (the label didn't position quite right).



a) Convert this to a DFA.

b) Describe in English the strings that are accepted by these automata,

3. Is the language of strings of 0s and 1s that have different numbers of 0s than 1s regular? For example 001, 0101010 and 111 are all in this language. Either prove the language is regular or prove that it isn't.

4. Consider the language $\{0^n(0^m1^m)^n \mid n > 0, m>0\}$ Just to be clear, strings in this language start with n 0s. They then have n groups, where each group consists of some number of 0s followed by the same number of 1s. For example, 000001101000111 is in this language.

   a) Give a grammar for this language.

   b) Give a parse tree for the derivation of 000001101000111 with your grammar.

5. Show that the language $\{0^n 1^m 2^n \mid n > 0, m < n\}$ is not context-free.

6. Describe a TM that takes as input a string of n 0s and halts with 2^n 0s on its tape. You can use as many tapes as you want, though the number of tapes needs to be a constant and can't depend on n. It is not necessary to give all of the machine's transitions; just break this down to simple steps that can clearly be performed by a Turing Machine.

7. Let $\mathcal{L}_{\text{hippy-dippy}}$ be the set of encodings of Turing Machines that accept all strings. Our friend Happy (actually, his encoding) is a member of $\mathcal{L}_{\text{hippy-dippy}}$. The complement of $\mathcal{L}_{\text{hippy-dippy}}$ is $\mathcal{L}_{\text{skeptical}}$, the set of Turing Machines that fail to accept at least one string. Rice's Theorem tells us that neither of these sets is Recursive. Are either of them Recursively Enumerable? You can use facts we proved in class about the Diagonal language, the Universal language, the Halting language, and the Empty and Non-Empty languages (and the complements of any of these). Anything else you use you need to prove.
   a. Either prove that $\mathcal{L}_{\text{hippy-dippy}}$ is Recursively Enumerable or prove it isn't.

   b. Either prove that $\mathcal{L}_{\text{skeptical}}$ is Recursively Enumerable or prove it isn't.

8. Explain in English what Cook's Theorem (aka the Cook-Levin Theorem) means, without using the terms $\mathcal{P}$, $\mathcal{NP}$, NP-Complete and NP-Hard

You can use this page as extra space for any problem.

Please write and sign the Honor Pledge when you have finished the exam. If you didn't take the exam with the rest of the class also write your starting and stopping times.